Problem 1.13

A mole is approximately the number of protons in a gram of protons. The mass of a neutron is about the same as the mass of a proton, while the mass of an electron is usually negligible in comparison, so if you know the total number of protons and neutrons in a molecule (i.e., its "atomic mass"), you know the approximate mass (in grams) of a mole of these molecules.* Referring to the periodic table at the back of this book, find the mass of a mole of each of the following: water, nitrogen (N_2), lead, quartz (SiO_2).

Solution

This is a trick question because isotopes exist: The atomic number for a given element tells how many protons each atom has, but atoms can differ in the number of neutrons they have. The average atomic mass, listed at the bottom of each block in the periodic table on page 403, takes into account the masses of the naturally occuring isotopes and their fractional abundances and should be used to compute the molar mass of each molecule.

Water (H_2O)

The average atomic mass of hydrogen is 1.00794 amu, and the average atomic mass of oxygen is 15.9994 amu. There are two hydrogen atoms and one oxygen atom in each water molecule.

$$\left(2\times 1.00794\,\cancel{\text{amu}}\times\frac{1\text{ g}}{6.02\times 10^{23}\,\cancel{\text{amu}}}\times\frac{6.02\times 10^{23}\text{ atoms}}{1\text{ mol}}\right)$$
$$+\left(1\times 15.9994\,\cancel{\text{amu}}\times\frac{1\text{ g}}{6.02\times 10^{23}\,\cancel{\text{amu}}}\times\frac{6.02\times 10^{23}\text{ atoms}}{1\text{ mol}}\right)\approx 18.0153\ \frac{\text{g}}{\text{mol}}$$

Therefore, the mass of one mole of water is about 18.0153 g.

Nitrogen (N_2)

The average atomic mass of nitrogen is 14.00674 amu, and there are two nitrogen atoms in each nitrogen molecule.

$$\left(2\times 14.00674\,\cancel{\text{amu}}\times\frac{1\text{ g}}{6.02\times 10^{23}\,\cancel{\text{amu}}}\times\frac{6.02\times 10^{23}\text{ atoms}}{1\text{ mol}}\right)=28.01348\ \frac{\text{g}}{\text{mol}}$$

Therefore, the mass of one mole of nitrogen (N_2) is about 28.01348 g.

*The precise definition of a mole is the number of carbon-12 atoms in 12 grams of carbon-12. The **atomic mass** of a substance is then the mass, in grams, or exactly one mole of that substance. Masses of *individual* atoms and molecules are often given in **atomic mass units**, abbreviated "u", where 1 u is defined as exactly 1/12 the mass of a carbon-12 atom. The mass of an isolated proton is actually slightly greater than 1 u, while the mass of an isolated neutron is slightly greater still. But in this problem, as in most thermal physics calculations, it's fine to round atomic masses to the nearest integer, which amounts to counting the total number of protons and neutrons.

Lead (Pb)

The average atomic mass of lead is 207.2 amu.

$$\left(207.2\,\cancel{\text{amu}} \times \frac{1\text{ g}}{6.02\times10^{23}\,\cancel{\text{amu}}} \times \frac{6.02\times10^{23}\text{ atoms}}{1\text{ mol}}\right) = 207.2\,\frac{\text{g}}{\text{mol}}$$

Therefore, the mass of one mole of lead is about 207.2 g.

Quartz (SiO_2)

The average atomic mass of silicon is 28.0855 amu, and the average atomic mass of oxygen is 15.9994 amu. There are two oxygen atoms and one silicon atom in each quartz molecule.

$$\left(1\times 28.0855\,\cancel{\text{amu}} \times \frac{1\text{ g}}{6.02\times10^{23}\,\cancel{\text{amu}}} \times \frac{6.02\times10^{23}\text{ atoms}}{1\text{ mol}}\right) + \left(2\times 15.9994\,\cancel{\text{amu}} \times \frac{1\text{ g}}{6.02\times10^{23}\,\cancel{\text{amu}}} \times \frac{6.02\times10^{23}\text{ atoms}}{1\text{ mol}}\right) = 60.0843\,\frac{\text{g}}{\text{mol}}$$

Therefore, the mass of one mole of quartz (SiO_2) is about 60.0843 g.